A DIFFERENTIAL METHOD OF DETERMINING THE THERMAL PROPERTIES OF MATERIALS

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A method for determining the thermal properties of materials is described, which enables one to distinguish small differences in the thermal properties of two specimens. Only two galvanometer readings need to be taken. The method belongs to the category of "electrical measurements of nonelectrical quantities."

A plane-parallel layer of the material under test M and a standard plate A have a common heat receiver B, separated into axial parts by a heat-insulating layer E (Fig. 1).

Two galvanometers g_I and g_{II}, with rheostats R_I and R_{II} are connected in the circuit of two differential thermocouples, as shown in Fig. 1. If the system AMB is brought into contact with the heater H at constant temperature t_H, the increase in the relative temperature $\theta = t/t_H$ with time τ at the points c_4 and c_3 is given respectively by the following expressions [1, 2]:

$$\theta = (1+\alpha)(\operatorname{erfc} y - \alpha \operatorname{erfc} 3y + \alpha^2 \operatorname{erfc} 5y \ldots), \quad (1)$$

$$\theta_A = (1+\alpha_A)(\operatorname{erfc} y_A - \alpha_A \operatorname{erfc} 3y_A + \alpha_A^2 \operatorname{erfc} 5y_A \ldots), \quad (2)$$

TABLE 1. Values of $y' = f_1(y'/y'')$ and $\varepsilon = f_2(y'/y'')$

y'/y''	y'	ε	y'/y''	y'	ε	y'/y''	y'	ε
			for $1-\theta'=0,90$; $1-\theta''=0,75$					
1,34	1,279	2,400	1,60	1,028	0,518	1,95	0,850	0,278
1,36	1,240	1,700	1,65	0,995	0,460	2,00	0,828	0,262
1,38	1,218	1,400	1,70	0,968	0,415	2,05	0,806	0,242
1,40	1,194	1,201	1,75	0,945	0,376	2,10	0,779	0,228
1,45	1,141	0,896	1,80	0,922	0,348	2,15	0,749	0,216
1,50	1,095	0,710	1,85	0,899	0,320	2,20	0,718	0,202
1,55	1,064	0,609	1,90	0,873	0,298	2,25	0,686	0,190
			for $1-\theta'=0,75$; $1-\theta''=0,50$					
1,42	0,989	3,45	1,75	0,793	0,915	2,20	0,598	0,453
1,44	0,972	2,86	1,80	0,774	0,836	2,25	0,574	0,420
1,46	0,956	2,44	1,85	0,752	0,764	2,30	0,546	0,386
1,48	0,943	2,18	1,90	0,731	0,707	2,35	0,520	0,359
1,50	0,926	1,92	1,95	0,712	0,650	2,40	0,491	0,330
1,55	0,896	1,563	2,00	0,690	0,605	2,45	0,455	0,300
1,60	0,864	1,308	2,05	0,670	0,562	2,50	0,417	0,267
1,65	0,838	1,132	2,10	0,647	0,524	2,55	0,377	0,236
1,70	0,814	1,004	2,15	0,623	0,487	2,60	0,333	0,201
			for $1-\theta'=0,50$; $1-\theta''=0,25$					
1,50	0,695	3,560	1,80	0,584	1,618	2,10	0,479	1,017
1,55	0,678	2,831	1,85	0,567	1,486	2,15	0,463	0,943
1,60	0,659	2,422	1,90	0,551	1,383	2,20	0,440	0,878
1,65	0,641	2,158	1,95	0,530	1,278	2,25	0,421	0,810
1,70	0,621	1,950	2,00	0,516	1,180	2,30	0,396	0,743
1,75	0,602	1,762	2,05	0,500	1,098	2,35	0,374	0,680

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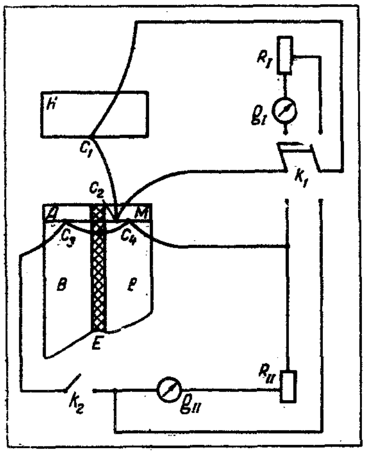

Fig. 1

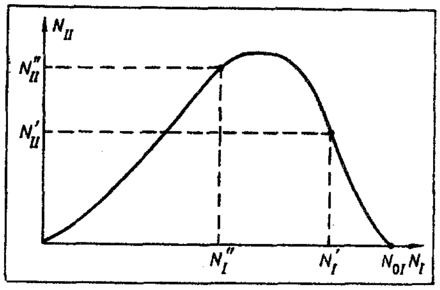

Fig. 2

Fig. 1. Sketch of the laboratory arrangement (the switch k_1 can be used to switch galvanometers g_I and g_{II} when choosing the values of N_{0I} and N_{0II}, and to connect galvanometer g_I into the operating position; the switch k_2 connects the galvanometer g_{II} into the operating position).

Fig. 2. Graph of N_{II} against N_I for the galvanometers g_{II} and g_I (N'_I and N''_I are assigned scale divisions of the galvanometer g_I, and N''_I and N''_{II} are measured scale divisions of galvometer g_{II}).

where

$$\alpha = \frac{\varepsilon - 1}{\varepsilon + 1}; \quad \varepsilon = \frac{\lambda}{b\sqrt{a}}; \quad b = \frac{\lambda_B}{\sqrt{a_B}};$$
$$\alpha_A = \frac{\varepsilon_A - 1}{\varepsilon_A + 1}; \quad \varepsilon_A = \frac{\lambda_A}{b\sqrt{a_A}}; \tag{3}$$
$$y = \frac{h_M}{2\sqrt{a\tau}}; \quad y_A = \frac{h_A}{2\sqrt{a_A\tau}}.$$

At the same instant of time $y_A/y = (h_A/h_M)\sqrt{a/a_A}$. If $h_A = h_M = h$, we have

$$y_A/y = \sqrt{\frac{a}{a_A}}. \tag{4}$$

The relation between the quantities y, α, and θ, expressed by Eqs. (1) and (2), can be represented by a table of "nodal points" [2].

The galvanometer g_I measures the relative temperature θ at the boundary of the media M and B at the point c_2. The galvonometer g_{II} measures the difference $\Delta\theta = \theta_A - \theta$.

The quantities θ and $\Delta\theta$ are related to the readings N_I and N_{II} of galvanometers g_I and g_{II} by the following equations:.

$$\theta = 1 - N_I/N_{0I}; \quad \Delta\theta = N_{II}/N_{0II}.$$

Here N_{0I} and N_{0II} are the readings of galvanometers g_I and g_{II} corresponding to a temperature difference $t_H - t_0$, where t_0 is the initial temperature of the system AMB. Using the rheostats R_I and R_{II} one can set the values of N_{0I} and N_{0II} which are convenient for the experiment.

The thermal characteristics are measured as follows. When the galvanometer g_I shows assigned divisions N'_I and N''_I, readings N'_{II} and N''_{II} are taken on the galvanometer g_{II} (Fig. 2). Since the quantities $\theta' = 1 - N'_I/N_{0I}$, $\theta'' = 1 - N''_I/N_{0I}$, $\Delta\theta' = N'_{II}/N_{0II}$ and $\Delta\theta'' = N''_{II}/N_{0II}$ are known, we also know the values $\theta'_A = \theta' + \Delta\theta'$ and $\theta''_A = \theta'' + \Delta\theta''$. The table of "nodal points" enables us for a given α_A to obtain $y'_A = f(\theta'_A)$ and $y''_A = f(\theta''_A)$.

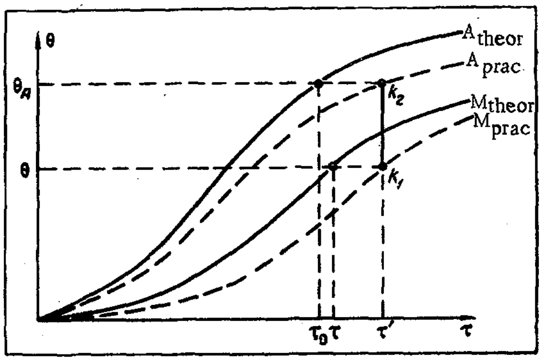

Fig. 3. Graph of θ against τ for theoretical and practical heating processes in the materials A and M (the continuous lines represent the theoretical process, and the dashed lines represent the practical process)

If the material of plate A and the heat receiver B are the same, then $\alpha_A = 0$, and Eq. (2) takes the simpler form

$$\theta_A = \operatorname{erfc} y_A = 1 - \operatorname{erf} y_A$$

or

$$\operatorname{erf} y_A = 1 - \theta_A. \qquad (5)$$

In this case the table of "nodal points" is not required, since it is easy to calculate y'_A and y''_A from the corresponding values of θ'_A and θ''_A, by using well-known tables of the probability integral [3].

It can be seen from expressions (3) and (4), that at the same instants of time

$$y'_A/y''_A = y'/y'',$$

where y' and y" are the arguments of Eq. (1) corresponding to the values θ' and θ''.

The values of the ratio y'/y'' for fixed values of θ' and θ'' completely determines the parameters ε and y' [4, 5], which occur in the theoretical relations

$$a = \frac{h_M^2}{4y'^2 \tau_1} \text{ and } \lambda = b\varepsilon\sqrt{a}.$$

But since $a_A = h_A^2/4y'^2_A\tau_1$, we have $a = a_A(h_M y'_A/h_A y')^2$.

For $h_M = h_A$ we obtain

$$a = a_A\left(\frac{y'_A}{y'}\right)^2. \qquad (6)$$

Further since $\lambda_A = b\varepsilon_A\sqrt{a_A}$, we have $\lambda = \lambda_A(\varepsilon/\varepsilon_A)\cdot(y'_A/y')$. If the material of plate A and the heat receiver B are the same, we have $\varepsilon_A = 1$ and

$$\lambda = \lambda_A \varepsilon \frac{y'_A}{y'}. \qquad (7)$$

Tables of $y' = f_1(y'/y'')$ and $\varepsilon = f_2(y'/y'')$ for the cases

1) $\theta' = 0.10$ and $\theta'' = 0.25$,

2) $\theta' = 0.25$ and $\theta'' = 0.50$,

3) $\theta' = 0.50$ and $\theta'' = 0.75$,

constructed on the basis of the table of "nodal points" are given in abbreviated form (see Table 1).

We will give an example of the calculation of the thermal characteristics from experimental data. The material investigated was petroleum jelly. The standard plate and the heat receiver were made of Plexiglas, the characteristics of which at room temperature are as follows: $a_A = 11.0\cdot10^{-8}$ m²/sec, and $\lambda_A = 0.173$ W/m·degree.

The condition $h_M = h_A = h$ was satisfied in the experiment.

The values $\theta' = 0.25$ and $\theta'' = 0.50$, set on galvanometer g_I, corresponded to the values $\Delta\theta' = 0.084$ and $\Delta\theta'' = 0.088$ measured on galvanometer g_{II}. Then

$$\theta'_A = \theta' + \Delta\theta' = 0.250 + 0.084 = 0.334,$$

$$\theta''_A = \theta'' + \Delta\theta'' = 0.500 + 0.088 = 0.588.$$

The values of the arguments y'_A and y''_A corresponding to these values of θ'_A and θ''_A, are found from Eq. (5)

$$\operatorname{erf} y'_A = 1 - \theta'_A = 1 - 0.334 = 0.666,$$

$$\operatorname{erf} y''_A = 1 - \theta''_A = 1 - 0.588 = 0.412.$$

Using tables of the probability integral [3], we find $y'_A = 0.683$ and $y''_A = 0.383$. Then $y'/y'' = y'_A/y''_A = 1.78$. From the tables of $y' = f_1(y'/y'')$ and $\varepsilon = f_2(y'/y'')$ for $\theta' = 0.25$ and $\theta'' = 0.50$ (see Table 1) we obtain $y' = 0.783$ and $\varepsilon = 0.873$.

Consequently

$$a = a_A(y'_A/y')^2 = 11 \cdot 10^{-8}\left(\frac{0.683}{0.783}\right)^2 \text{ m}^2/\text{sec},$$

$$= 11 \cdot 10^{-8}(0.874)^2 \text{ m}^2/\text{sec} = 8.40 \cdot 10^{-8} \text{ m}^2/\text{sec},$$

$$\lambda = \lambda_A \varepsilon \frac{y'_A}{y'} = 0.173 \cdot 0.873 \cdot 0.874 \text{ W/m}\cdot\text{degree} = 0.132 \text{ W/m}\cdot\text{degree}.$$

Observations are best made so as to use at the same time two or three tables for y' and ε, corresponding to the following values of θ' and θ'': (0.10; 0.25); (0.25; 0.50); (0.50; 0.75). In this case, we can obtain from a single experiment three independent values of the characteristics a and λ, the average of which will be the most probable value for the given experiment in the chosen temperature range.

The differential method is suitable for investigating materials which differ only slightly in their thermal properties from the chosen standard material. For example, the method is suitable for making measurements of the characteristics of solutions as a function of the concentration. In this case the standard plate can be either pure solvent, or a solution of known concentration.

The method also enables certain experimental errors to be eliminated.

In fact, because of errors which distort the results (leakage of heat, the presence of contact resistance, insufficient heating power, etc.), the actual rise in temperature at the points c_3 and c_4 may differ from the actual process $\theta = f(\tau)$ given by Eqs. (1) and (2).

Because of these errors the specified value of θ will be recorded not at the instant τ, but at the instant $\tau' > \tau$. At this instant τ' the measured difference $\Delta\theta$ $-k_1k_2$ (see Fig. 3) determines the quantity

$$\theta_A = \theta + \Delta\theta.$$

Since, to obtain the argument y_A we used the theoretical equation (2), the value of y_A obtained corresponds to the time $\tau_0 < \tau'$.

Assuming the values $\tau = \tau_0$ in the equations $y_A = h/2\sqrt{a_A\tau}$ and $y = h/2\sqrt{a\tau}$ to be the same, the quantity θ will relate not to the instant of time τ', but to the instant $\tau_0 < \tau'$.

If the delay due to the imperfection of the experiment is the same for both plates A and M, i.e., if $\tau' - \tau = \tau' - \tau_0$ (see Fig. 3), in this case $\tau_0 = \tau$, which is equivalent to eliminating the experimental error.

In the differential method, no time measurement is required. Also, the thickness of the material does not occur in the theoretical equations. The condition $h_A = h_M$ can be satisfied by combining a solid plate with a liquid or free-flowing material.

Only the readings of two galvanometers are required in this method. Consequently, the method belongs to the category of "electrical measurements of nonelectrical quantities," which enables the accuracy of thermal measurements to be increased.

LITERATURE CITED

1. A. V. Lykov, Theory of Heat Conduction [in Russian], Gostekhizdat (1952).
2. V. S. Vol'kenshtein, Proceedings of the Second All-Union Conference on Heat and Mass Transfer, Vol. 7, Minsk (1966).

3. B. I. Segal and K. A. Semendyaev, Mathematical Tables [in Russian], Gosizdat (1950).
4. V. S. Vol'kenshtein, Proceedings of the Lensovet Technological Institute, No. 32 (1954).
5. V. S. Vol'kenshtein and N. N. Medvedev, Inzh.-Fiz. Zh., 2, No. 10 (1959).